PML Absorbing Boundary Condition for Noncubic Cell Time-Domain Method

Norihiko Harada and Mitsuo Hano

***Abstract*— Conventional absorbing boundary conditions (ABC's) are used for cubic-cell finite-difference time-domain (FDTD) grids, but not in noncubic cell time-domain grids. We propose an algorithm to apply perfectly matched layer (PML) ABC to the noncubic cell time-domain method. The numerical experiments are conducted of the accuracy of the PML in a regular triangle cell grid versus standard second-order Mur and PML ABC's in a square-cell FDTD grid. The PML global error is about 10^{-3} of the former and about three times of the latter. According to the convergent characteristics this algorithm is equivalent to that of applying PML ABC to the FDTD method.**

***Index Terms*— Electromagnetic-field equations, FDTD, PML absorbing boundary conditions.**

I. Introduction

The finite-difference time-domain (FDTD) method, based on the traditional Yee-algorithm [1], has been successfully applied to the analysis of many problems of interest. However, the rectangular cells restrict the stair-casing approximation for the curved boundaries. A noncubic cell time-domain method [2] was suggested and reported as being effective for modeling of curved surface using prism cells of arbitrary shape. Each side of the cell corresponds to its normal polygon whose apexes are the circumcenter of proximal cells. Each side of the polygons also correspond to its normal face of a cell. The polygons of its Voronoi diagram [3] and the faces as finite elements are chosen as integral surfaces for Ampere's and Faraday's laws, respectively.

Many geometries of interest are defined in "open" regions where the spatial field computation domain is limited in size, so a suitable boundary condition of the outer perimeter of the domain must be used to simulate its extension to infinity. Mur introduced the simple and successful finite-difference scheme for the absorbing boundary condition (ABC) [4]. Berenger suggested a novel ABC for the FDTD method, regarded as a perfectly matched layer, and improved the performance relative to any earlier technique [5]. It has the exponential formula since FDTD has derivative formula for calculating magnetic and electric field components. The formula of Mur ABC is based on cubic-cell grids, so the formula is expected to be complex for applying the ABC to noncubic cell time-domain method.

Manuscript received May 14, 1997.

N. Harada is with the Department of Computer Science and Electronic Engineering, Tokuyama College of Technology, Kume Tokuyama 745, Japan.
M. Hano is with the Department of Electrical and Electronic Engineering, Faculty of Engineering, Yamaguchi University, Tokiwadai Ube 755, Japan.

Publisher Item Identifier S 1051-8207(97)06956-0.

In this letter we propose the perfectly matched layer (PML) formula for noncubic cell time-domain method, splitting magnetic field components in the absorbing boundary region with the possibility of assigning losses to the individual split field components. The formulation is simple and has no exponential formula in the lossy domain, since the updated equations are directly obtained from Ampere's and Faraday's laws.

II. Formulation

Maxwell's equations for the two-dimensional TM polarization case, where the relevant field components include E_z, B_x, and B_y, reduce to

$$\frac{1}{\mu_0}\nabla\times(B_x\mathbf{i}+B_y\mathbf{j})=\left(\epsilon_0\frac{\partial E_z}{\partial t}+\sigma E_z\right)\mathbf{k} \tag{1}$$

$$-\nabla\times(E_z\mathbf{k})=\frac{\partial}{\partial t}(B_x\mathbf{i}+B_y\mathbf{j})+\frac{\sigma^*}{\mu_0}(B_x\mathbf{i}+B_y\mathbf{j}) \tag{2}$$

where ϵ_0 and μ_0 are the free-space permittivity and permeability, the parameter σ denotes electric conductivity, and the parameter σ^* denotes magnetic loss. It is well known that if the following condition is satisfied:

$$\frac{\sigma}{\epsilon_0}=\frac{\sigma^*}{\mu_0} \tag{3}$$

the wave impedance of the lossy free-space medium equals that of lossless vacuum, and no reflection occurs when a plane wave propagates normally across an interface between a true vacuum and the lossy free-space medium. Using Ampere's and Faraday's law, (1) and (2) can be rewritten as

$$\frac{1}{\mu_0}\int_{S_a}(B_n\mathbf{n})\cdot d\mathbf{l}=\iint_{S_a}\left(\epsilon_0\frac{\partial E_z}{\partial t}+\sigma E_z\right)ds \tag{4}$$

$$-\int_{S_f}E_z\,dl=\iint_{S_f}\left(\frac{\partial B_n}{\partial t}+\frac{\sigma^*}{\mu_0}B_n\right)\mathbf{n}\cdot d\mathbf{s}. \tag{5}$$

Each of field components B_n in (4) and (5) can be split into individual components x and y. As a result, (4) and (5) become

$$\frac{1}{\mu_0}\int_{C_a}(B_{nx}\mathbf{i})\cdot d\mathbf{l}=\iint_{S_a}\left(\epsilon_0\frac{\partial E_{zy}}{\partial t}+\sigma_y E_{zy}\right)ds \tag{6}$$

$$\frac{1}{\mu_0}\int_{S_a}(B_{ny}\mathbf{j})\cdot d\mathbf{l}=\iint_{C_a}\left(\epsilon_0\frac{\partial E_{zx}}{\partial t}+\sigma_x E_{zx}\right)ds \tag{7}$$

$$-\int_{S_f}(E_{zy}+E_{zx})\,dl=\iint_{S_f}\left(\frac{\partial B_{nx}}{\partial t}+\frac{\sigma_y^*}{\mu_0}B_{nx}\right)\mathbf{i}\cdot d\mathbf{s}+\iint_{S_f}\left(\frac{\partial B_{ny}}{\partial t}+\frac{\sigma_x^*}{\mu_0}B_{ny}\right)\mathbf{j}\cdot d\mathbf{s}. \tag{8}$$

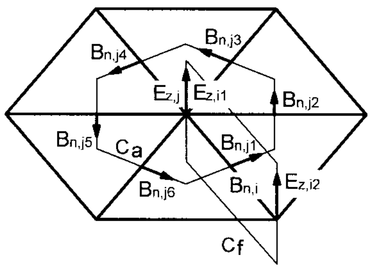

Fig. 1. Position of the electric and magnetic field vector components about regular triangle cells and chain-linked orthogonal contours in the noncubic cell for the two-dimensional TM case.

Fig. 1 shows the position of the electric and magnetic field vector components about regular triangle cells and chain-linked orthogonal contours in the noncubic cell for the two-dimensional TM case. From (6) to (8) discrete formulas are given as

$$E_{zy,j}^{t+1/2} = \left(\frac{\epsilon_0}{\Delta t} + \frac{\sigma_y}{2}\right)^{-1} \left[\frac{1}{S\mu_0}\sum_{k=1}^{6} B_{nx,jk}^{t}\theta_{x,jk}l + \left(\frac{\epsilon_0}{\Delta t} - \frac{\sigma_y}{2}\right)E_{zy,j}^{t-1/2}\right] \tag{9}$$

$$E_{zx,j}^{t+1/2} = \left(\frac{\epsilon_0}{\Delta t} + \frac{\sigma_x}{2}\right)^{-1} \left[\frac{1}{S\mu_0}\sum_{k=1}^{6} B_{ny,jk}^{t}\theta_{y,jk}l + \left(\frac{\epsilon_0}{\Delta t} - \frac{\sigma_x}{2}\right)E_{zx,j}^{t-1/2}\right] \tag{10}$$

$$E_{z,j} = E_{zx,j} + E_{zy,j} \tag{11}$$

$$B_{n,i}^{t} = \left(\frac{1}{\Delta t} + \frac{\sigma_y^*\theta_{x,t}^2 + \sigma_x^*\theta_{y,t}^2}{2\mu_0}\right)^{-1} \left[\frac{E_{n,i2}^{t-1/2} - E_{n,i1}^{t-1/2}}{\Delta x} + \left(\frac{1}{\Delta t} - \frac{\sigma_y^*\theta_{x,i}^2 + \sigma_x^*\theta_{y,i}^2}{2\mu_0}\right)B_{n,i}^{t-1}\right] \tag{12}$$

$$B_{nx,i}^{t} = B_{n,i}^{t}\theta_{xi}, \quad B_{ny,i}^{t} = B_{n,i}^{t}\theta_{yi} \tag{13}$$

$$\theta_x = \mathbf{i}\cdot\mathbf{n}, \quad \theta_y = \mathbf{j}\cdot\mathbf{n} \tag{14}$$

where S denotes the area within Ampere's integral surface, Δt is one time interval, θ_x is the inner product of unit vectors along x axis and normal to Faraday's integral surface, and θ_y is the same but along y axis.

III. Experiments

Fig. 2 shows the experimental setup for evaluating the absorbing boundary condition in the noncubic cell time-domain method. The simulation models of wave propagation in two-dimensional TM polarization case. The point source is a sinusoidal wave with a frequency of 2 [GHz]. Two simulations are computed simultaneously in order to calculate error caused by the PML medium. In one case waves propagate into a normal free-space computational zone surrounded by a PML backed by perfectly conducting (PEC) walls. In the other case waves propagate into a wider free-space stretched away to the outer PEC walls. The global error is defined as

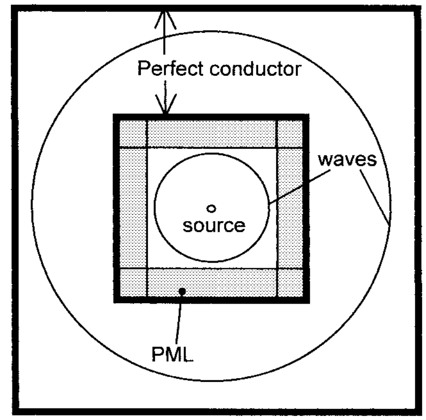

Fig. 2. Experimental setup for evaluating ABC in noncubic cell time-domain method.

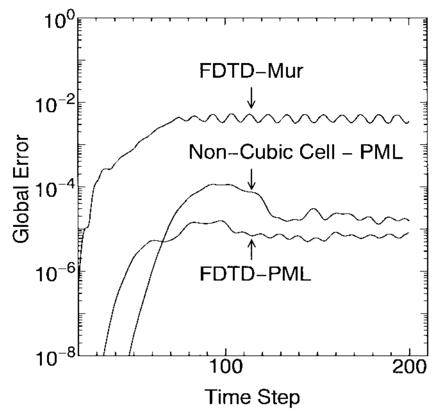

Fig. 3. The global error within the 60 × 20-cell lattices plotted as a function of time step number for three different methods.

$$\text{Global Error} = \frac{\sum_{\substack{|x|<x_{\max}\\|y|<y_{\max}}} (E_z - E_{z0})^2}{\sum_{\substack{|x|<x_{\max}\\|y|<y_{\max}}} E_{z0}^2}. \tag{15}$$

The parameters of numerical experiments are given as follows:

$$\Delta x = \frac{\lambda}{10}, \quad \Delta t = \frac{\Delta x}{2c} \tag{16}$$

$$\frac{\sigma_x}{\epsilon_0} = \frac{\sigma_x^*}{\mu_0}, \quad \frac{\sigma_y}{\epsilon_0} = \frac{\sigma_y^*}{\mu_0} \tag{17}$$

$$\sigma_y = \sigma_{y\max}\left(\frac{y - y_{\max}}{y_{\text{PEC}} - y_{\max}}\right)^4 \tag{18}$$

$$\sigma_x = \sigma_{x\max}\left(\frac{x - x_{\max}}{x_{\text{PEC}} - x_{\max}}\right)^4 \tag{19}$$

where Δx is the length of a side of a regular triangle.

In order to compare the error level of noncubic cell time-domain method using the PML in a regular triangle cell grid with that of FDTD simulation in a square cell grid using PML or second-order Mur ABC these three simulations were done with the same conditions. The PML thickness was 16 cells. $\sigma_{\max}/\epsilon_0$ was selected as 3.6 $\times 10^{10}$. For the calculation of lossless area, let $\sigma = 0$. For comparing the effects of incident angle of wave front to absorbing boundary among the three

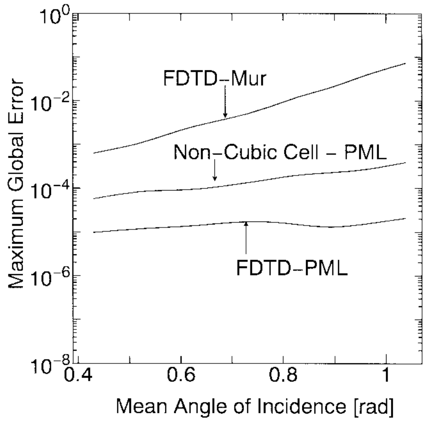

Fig. 4. Maximum global error versus the mean of the incident angles to the normal to absorbing boundary for three different methods.

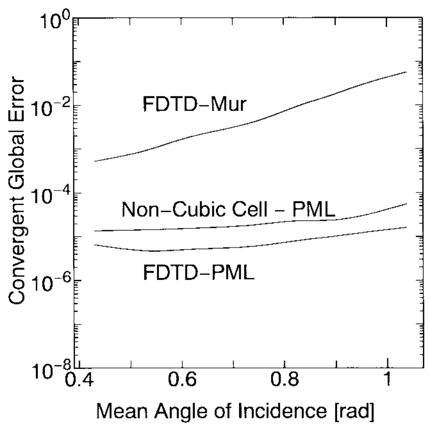

Fig. 5. Convergent global error versus the mean of the incident angle to the normal to absorbing boundary for three different methods.

different methods global error levels within 60×8 to 60×60-cell lattices were simulated, which region corresponds to the mean of incident angle 0.43 to 1.03 rad. In any case the reference region was within 200×200-cell lattices.

IV. RESULTS

Fig. 3 shows the global error within the 60×20-cell lattices plotted as a function of time step number for three different method. It was found that the error level of noncubic cell time-domain simulation with the PML ABC is ten times that of FDTD with PML ABC in maximum caused by earlier reflection at time step about 100. After that it is close to three times the error level of FDTD with PML ABC and 1/100th that of FDTD with Mur ABC. The influence of the wave front angle to the effectiveness of the absorbing boundary was studied. Fig. 4 shows maximum global error versus the mean of the incident angle to the normal to absorbing boundary. Fig. 5 shows convergent global error with the mean angle.

In convergence characteristics, the FDTD and noncubic cell time-domain methods are very similar to each other. It was found that the convergent error level of the noncubic time-domain method with the PML ABC is 1/1000th of the FDTD, with a Mur ABC with small dependence on the incident angle at a mean incident angle of 1 rad.

V. CONCLUSION

Applying the PML ABC for the noncubic cell time-domain method has been proposed. Using regular triangle cells the error level has small dependence on incident angle, similar to that of the FDTD with PML ABC, except in time steps when early reflection occurs. However, in any time steps the error level of noncubic cell time-domain simulation with the PML ABC is 1/10th to 1/1000th that of FDTD method with Mur ABC. According to the convergent characteristics of small dependence on incident angles, this algorithm is equivalent to that of applying PML ABC to the FDTD method. The method proposed in this letter is easily extendable to a three-dimensional problem with prism elements.

REFERENCES

[1] K. S. Yee, "Numerical solution of initial boundary value problems involving Maxwell's equations in isotropic media," *IEEE Trans. Antennas Propagat.*, vol. AP-14, pp. 302–307, May 1966.
[2] M. Hano and T. Itoh, "Three-dimensional time-domain method for solving Maxwell's equations based on circumcenters of elements," *IEEE Trans. Magn.*, vol. 32, pp. 946–949, May 1996.
[3] K. Sugihara and M. Iri, "Construction of the Voronoi diagram for one million generators in single-precision arithmetic," *Proc. IEEE*, vol. 80, pp. 1471–1484, Sept. 1992.
[4] G. Mur, "Absorbing boundary conditions for the finite-difference approximation of the time-domain electromagnetic field equations," *IEEE Trans. Electromagn. Compat.*, vol. 23, pp. 377–382, 1981.
[5] J.-P. Berenger, "A perfectly matched layer for the absorption of electromagnetic waves," *J. Comput. Phys.*, vol. 114, pp. 185–200, 1994.